

Active Tendon Control of Large Trusses

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The use of tension cables to stiffen and control precision trusses as needed for future interferometric missions is investigated. A strategy for damping cable structures with active tendons is presented. Each tendon consists of a displacement actuator (piezoelectric in this case) collocated with a force sensor; the local control law consists of an integral force feedback, which has guaranteed stability if we assume perfect actuator and sensor dynamics. Then an approximate linear theory that allows one to predict the closed-loop poles of a cable structure with a root locus technique is developed; the methodology is applied numerically to a model of the Jet Propulsion Laboratory Micro-Precision Interferometer. Finally, a laboratory experiment on a guyed truss with three active tendons is described, and the experimental results are compared with the numerical predictions.

I. Introduction

IN recent years, cable structures have had their most spectacular applications in civil engineering, particularly in large cable-stayed bridges. (The main spans of recent cable-stayed bridges approach 1000 m in length.) Cable structures have also been used on a smaller scale in space applications; a 10-m deployable mesh antenna was flown by the Russians in 1979, and a high-precision mesh antenna system with an equivalent aperture of 8 m was recently flown in Japan.¹ The concept of tension truss has many advantages, such as low weight and deployability; besides, it leads naturally to shape control and reconfiguration by changing the static tension in the cables.² The present study advocates the idea that tension cables can be used for the triple role of 1) lightweight stiffening, 2) shape control, and 3) active damping of vibrations. (It will be shown that the damping and stiffening effects are not independent.) The paper is focused on the use of active tendons for vibration damping of trusses with guyed cables. It is organized as follows: Sec. II summarizes the main results concerning the damping with active tendons. Section III explains how the closed-loop performances can be evaluated with a root locus technique; the theoretical results are confirmed by a numerical experiment. Section IV applies the methodology to a model of the Jet Propulsion Laboratory Micro-Precision Interferometer (JPL MPI), and Sec. V describes a laboratory experiment with a guyed truss. The nonlinear modeling aspects of cable structures are not addressed in detail in this paper; they were considered earlier.^{3,4}

II. Active Damping of Cable Structures

A. Tendon Control of Strings and Cables

The mechanism by which an active tendon can extract energy from a string or cable is explained in Fig. 1 through a simplified model assuming only one mode (Rayleigh–Ritz) in situations of increasing complexity. The simplest case is that of a linear string with constant tension T_0 (Fig. 1a); the equation becomes nonlinear when the effect of stretching is added (cubic nonlinearity). In Fig. 1b, a moving support is added; the input u of this active tendon produces a parametric excitation, which is the only way one can control a string with this type of actuator. The difference between a string and a cable (Fig. 1c) is the effect of gravity, which produces sag.

In this case, the equations of motion in the gravity plane and in the plane orthogonal to it are no longer the same, and they are coupled. In the gravity plane, the active tendon control u still appears explicitly as a parametric excitation, but it also appears as an inertia term $-\alpha_c \ddot{u}$ whose coefficient α_c depends on the sag of the cable; even for cables with moderate sag (e.g., sag-to-length ratio of 1% or more), this contribution becomes significant and constitutes the dominant control term of the equation. In contrast, in the out-of-plane equation (y coordinate), the tendon control u appears explicitly only through the parametric excitation, as for the string.

The use of the parametric excitation to damp the transverse vibration of a string was explored by Chen⁵; the same strategy was used to control the out-of-plane vibration of cables by Fujino et al.,⁶ who also investigated the use of the inertia term for active damping of cables in the gravity plane.⁷ These attempts used noncollocated feedback of the transverse amplitude; they worked well when the interaction of the cable with the supporting structure was weak, but they became unstable when the interaction was strong. In contrast, the approach followed in the present study, which is based on collocated actuator-sensor pairs, is guaranteed to stabilize all of the states that are controllable and observable.

B. Control Strategy

It is widely accepted that the active damping of linear structures is much simplified if one uses collocated actuator-sensor pairs⁸; for nonlinear systems, this configuration is still quite attractive because there exist control laws that are guaranteed to remove energy from the structure. The direct velocity feedback is an example of such “energy absorbing” control. When using a displacement actuator (active tendon) and a force sensor, the (positive) integral force feedback

$$u = g \int T \, dt \quad (1)$$

(refer to Fig. 2a for notations) also belongs to this class because the power flow from the control system is $W = -T\dot{u} = -gT^2$. This control law was already applied to the active damping of a truss structure in a previous study,⁹ and it is quite remarkable that it also applies to nonlinear structures; all of the states that are controllable and observable are asymptotically stable for any value of g (infinite gain margin).

C. Experiment

The foregoing theoretical results have been confirmed experimentally with a laboratory-scale cable structure similar to that represented schematically in Fig. 2a, where the active tendon consisted of a piezoelectric actuator.³ Figure 2b shows the experimental frequency response between a force applied to the structure and its acceleration; also shown is the free response of the structure with and without control. We see that the control system brings a substantial amount of damping to the system without

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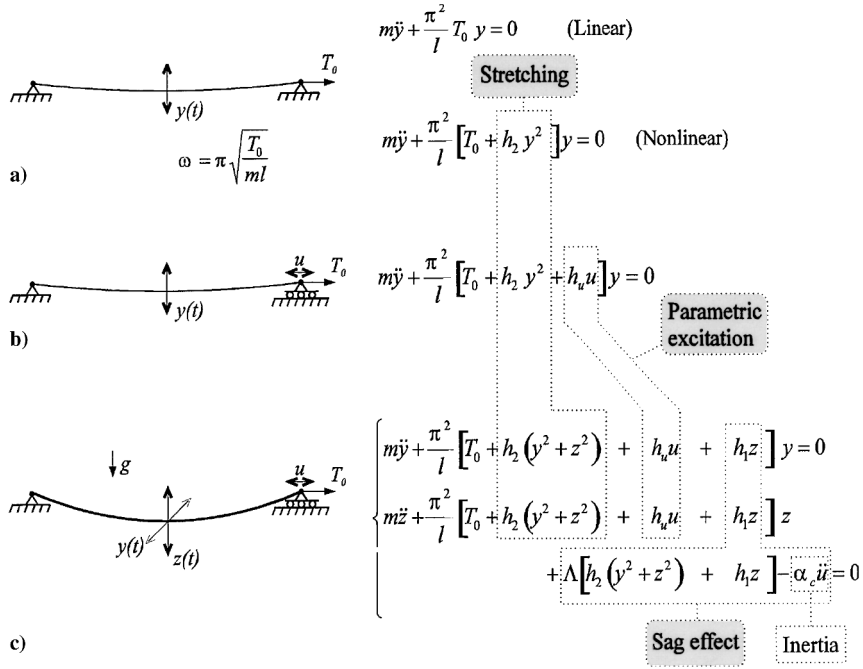


Fig. 1 Mechanism of active tendon control of strings and cables.

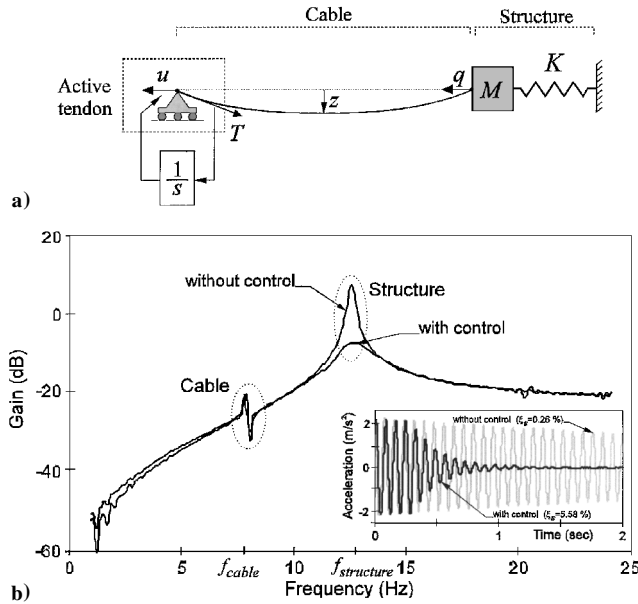


Fig. 2 Active damping of cable structures.

destabilizing the cable (theoretically, the control system does indeed bring a small amount of damping to the cable, which depends on the sag); this behavior is maintained at the parametric resonance, when the natural frequency of the structure is twice that of the cable.

D. Decentralized Control

The foregoing approach can readily be extended to the decentralized control of a structure with several active cables, each tendon working for itself with a local feedback following Eq. (1). This statement was verified experimentally on a T structure controlled with two cables.⁴ It is important to point out that the concept of active tendon control of cable structures does not require that all of the cables be active; on the contrary, the control system would normally involve only a small set of cables judiciously selected. The next section describes an approximate linear theory to predict the performance of the control system and provides design guidelines for selecting the active cables.

III. Prediction of the Closed-Loop Poles

A. Analytical Formulas

If we assume that the dynamics of the active cables can be neglected and that their interaction with the structure is restricted to the tension in the cables, it is possible to develop an approximate linear theory of the closed-loop system. The governing equation for the structure (neglecting the natural damping) is

$$M\ddot{x} + Kx = -BT \quad (2)$$

where M and K refer to the structure without the active cables (but with all of the passive ones acting as bars), T is the vector of the tension in the guy cables, and B is the influence matrix relating the local coordinate systems of the active cables to the global coordinates. If we neglect the active cable dynamics, their tension is given by

$$T = K_c(B^T x - \delta) \quad (3)$$

where $K_c = \text{diag}(h_c)$ is the diagonal matrix containing the stiffnesses of the active cables, $B^T x$ are the relative displacements of the extremities of the cables projected on the chord lines, and δ contains the active displacements of the tendons. The same matrix B appears in Eqs. (2) and (3) because the displacement actuator δ and the force sensor T are collocated. The decentralized control relates the control input δ to the sensor output T with a diagonal matrix; following Eq. (1), we adopt

$$\delta = (g/s)K_c^{-1}T \quad (4)$$

In this equation, $K_c^{-1}T$ represents the elastic extension of the active cables; the gain g is the same for all channels. Combining Eqs. (2)–(4), we obtain the closed-loop equation (in Laplace variable)

$$\{Ms^2 + (K + BK_c B^T) - [g/(s + g)]BK_c B^T\}x = 0 \quad (5)$$

From this equation, it is readily observed that, as $g \rightarrow \infty$, the dynamics of the closed-loop system converges to $[Ms^2 + K]x = 0$, which is the governing equation of the original structure without the active cables [see Eq. (2)]. Thus, the transmission zeros of the control system coincide with the poles of the structure where the active cables have been removed. If we assume that the mode shapes are not changed substantially by the control system (the validity of this assumption will be assessed in Sec. IV), it can be shown that, for each mode, the closed-loop poles follow a root locus. To show this,

let us denote by $\omega = \text{diag}(\omega_i)$ the diagonal matrix of natural frequencies of the structure without the active cables and by $\Omega = \text{diag}(\Omega_i)$ the matrix of natural frequencies when the active cables are in place; if the mode shapes are normalized according to $\Phi^T M \Phi = I$, we can project Eq. (5) in modal coordinates; we get

$$\{s^2 I + \Phi^T (K + B K_c B^T) \Phi - [g/(s + g)] \Phi^T B K_c B^T \Phi\} Z = 0 \quad (6)$$

From the assumption that the mode shapes are little changed by the presence of the active cables, we have

$$\Phi^T (K + B K_c B^T) \Phi = \Omega^2, \quad \Phi^T K \Phi = \omega^2 \quad (7)$$

so that Eq. (6) constitutes a set of decoupled equations:

$$s^2 I + \Omega^2 - [g/(s + g)](\Omega^2 - \omega^2) = 0$$

or equivalently

$$s(s^2 I + \Omega^2) + g(s^2 I + \omega^2) = 0 \quad (8)$$

Equation (8) shows that each mode follows the root locus corresponding to the open-loop transfer function

$$G(s) = g \frac{(s^2 + \omega_i^2)}{s(s^2 + \Omega_i^2)} \quad (9)$$

Thus, the closed-loop poles go from the open-loop poles at $\pm j\Omega_i$ for $g = 0$ to the open-loop zeros at $\pm j\omega_i$ for $g \rightarrow \infty$ (Fig. 3). All of these loops are always stable, and the maximum damping is

$$\xi_i^{\max} = (\Omega_i - \omega_i)/2\Omega_i \quad (10)$$

obtained for $g = \Omega_i \sqrt{(\Omega_i/\omega_i)}$. The root locus of Fig. 3 shows clearly that the depth of the loop in the left half plane is controlled by the frequency difference $\Omega_i - \omega_i$. This means that, to be efficient, the set of active cables must be sized and located in such a way that the presence of the active cables provides a substantial increase in the natural frequency of the modes that we wish to control. Upon computing the sensitivity derivative of the closed-loop poles, it can be further established that, for small gains, the modal damping ratio resulting from the active tendon control is given by

$$\xi_i \approx g v_i / 2\Omega_i \quad (11)$$

where $v_i = (\Omega_i^2 - \omega_i^2)/\Omega_i^2$ is the modal fraction of strain energy in the active cables. Equations (9–11) can be used very conveniently in the design of actively controlled cable structures.

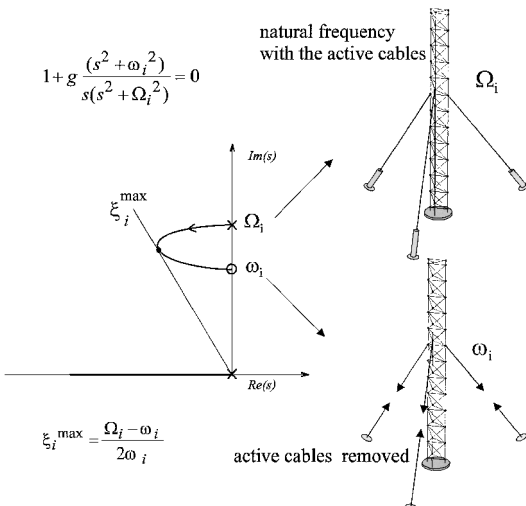


Fig. 3 Root locus of the closed-loop poles of a cable structure.

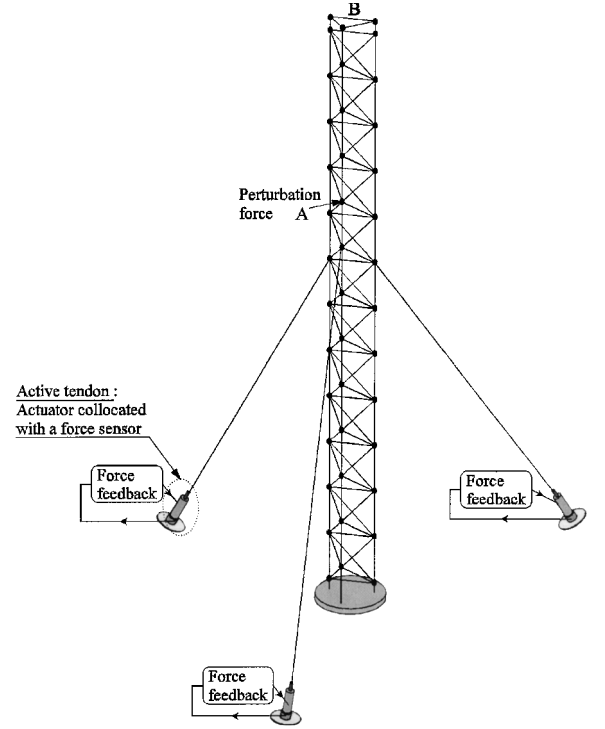


Fig. 4 Guyed truss.

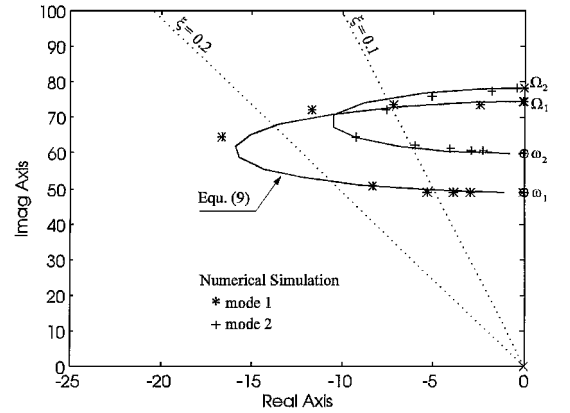


Fig. 5 Comparison of the root locus with the numerical simulation.

B. Numerical Experiment

To assess the accuracy of the linear predictions of the closed-loop poles, consider the first two modes of the guyed truss of Fig. 4. The structure consists of a 12-bay truss ($h = 1.7$ m) with three guyed cables made of steel, of 1-mm diam, connected to active tendons located 1 m from the base of the truss. A decentralized control is applied with the same gain for the three control loops. Figure 5 compares the prediction of the linear theory with a numerical experiment based on a nonlinear model including a full representation of the cable dynamics.³ The poles corresponding to the numerical dynamics are evaluated with a classical identification technique. The agreement is quite good. Further evidence of the quality of the closed-loop poles estimate will be given in Sec. V.

IV. Application to the JPL MPI Testbed

To illustrate the application of the control strategy to the damping of a large truss, let us consider the JPL MPI.¹⁰ The first three flexible modes are displayed in Fig. 6. We investigate the possibility of stiffness augmentation and active damping of these modes with a set of three active tendons acting on Kevlar cables of 2-mm diam, connected as indicated in Fig. 7 (Kevlar properties: $E = 130$ GPa, $\rho = 1500$ kg/m³, tensile strength $\sigma_y = 2.8$ GPa). The tension in the cables is taken in such a way that the operating stress is one-tenth of the tensile strength ($\sigma_0 = \sigma_y/10$), $T_0 = 879$ N. This

Table 1 Natural frequencies (rad/s) of the first flexible modes of the JPL MPI testbed, with and without the cables

i	ω_i	Ω_i
7	51.4	74.6
8	76.4	101
9	83.3	106.4

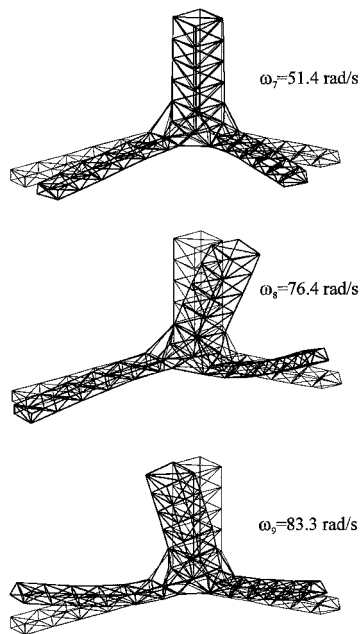


Fig. 6 Shape of the three global flexible modes (7–9).

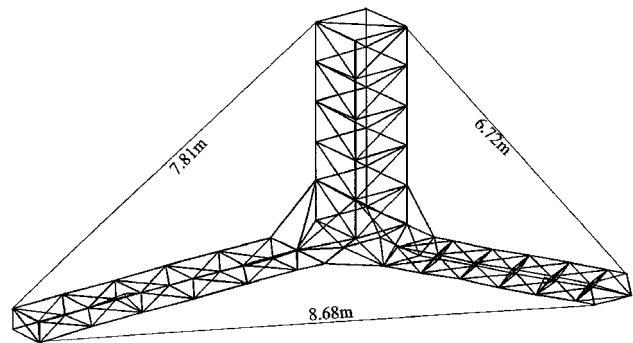


Fig. 7 Proposed location of the active cables in the JPL MPI testbed.

leads to natural frequencies of the three cables of, respectively, 201, 173, and 156 rad/s. The global added mass for the three cables is only 110 g (not including the active tendons and the control system). The natural frequencies of the first three modes with and without the cables are reported in Table 1; the root loci of the three global flexible modes as functions of the control gain g are represented in Fig. 8. The full lines refer to the approximate solution (9); the dotted lines correspond to the coupled-modes solution of Eq. (6). The two sets of curves are almost superimposed. Larger differences are observed if the cable diameter is increased; for $g = 116$ rad/s, the modal damping ratios are $\xi_7 = 0.21$, $\xi_8 = 0.16$, and $\xi_9 = 0.14$.

V. Experimental Results

An experiment (Fig. 9) has been conducted with a guyed truss very similar to that used in the numerical simulation of Fig. 4, except that the guyed cables are made of a synthetic fiber Dynema of 1-mm diam; the tension is adjusted to achieve a cable frequency larger than 500 rad/s.

The design of the active tendon is shown in Fig. 10; the amplification ratio of the lever arm is 3, leading to a maximum stroke of

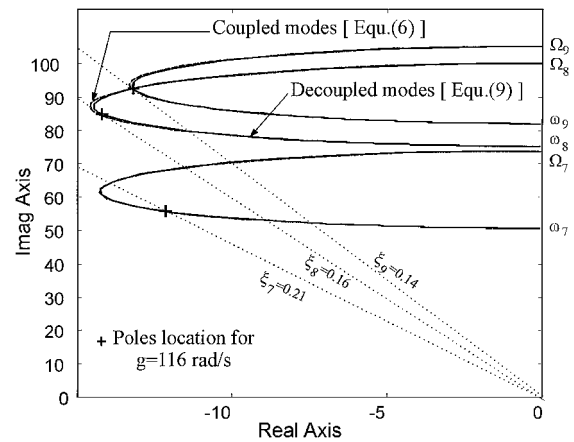


Fig. 8 Root locus of the three global flexible modes as functions of the control gain.

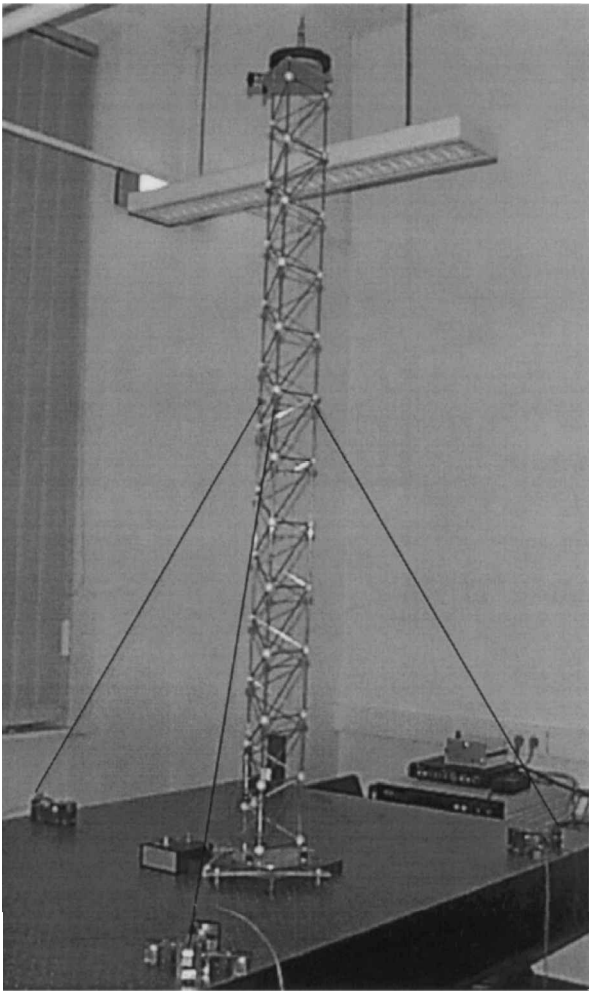


Fig. 9 Experimental setup.

about 150μ . The natural frequencies with and without the active cables (respectively, Ω_i and ω_i) are given in Table 2.

Figure 11 shows the root locus predicted by the linear model together with the experimental results for various values of the gain; only the upper part of the loops is available experimentally because the control gain is limited by the saturation due to the finite stroke of the actuators. The agreement between the experimental results and the linear predictions is quite good.

The guyed truss is actually the same as the one used in a previous study on active damping of trusses⁹; it is provided with two active struts near its base (visible in Fig. 9); each active strut consists of a piezoelectric actuator collocated with a force sensor. Figure 12

Table 2 Natural frequencies (rad/s) of the guyed truss, with and without the cables

i	ω_i	Ω_i
1	53.8	67.9
2	66	78.9

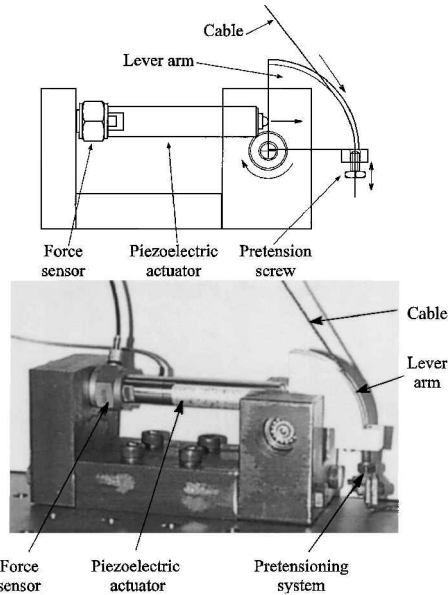


Fig. 10 Design of the active tendon.

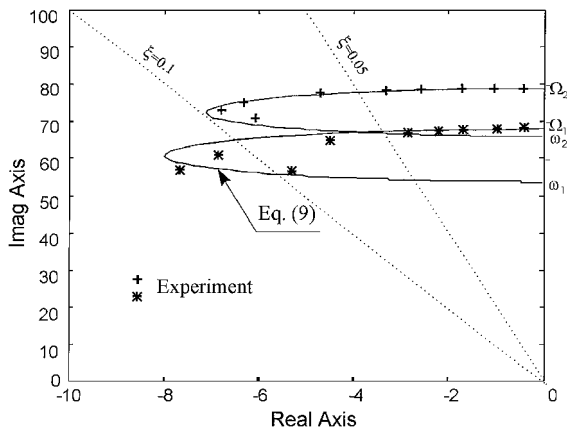


Fig. 11 Comparison of the root locus with the experiment.

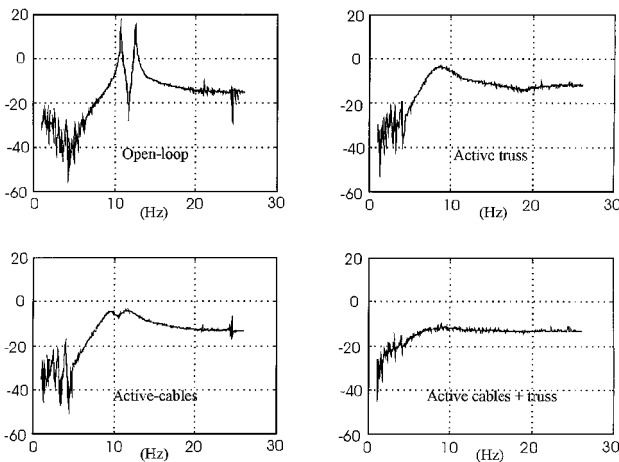


Fig. 12 Typical frequency response functions of the guyed truss for various control conditions.

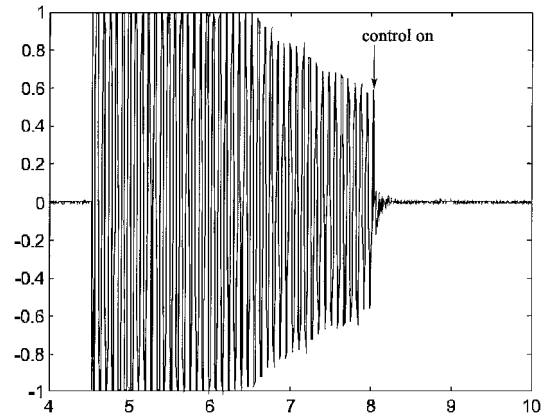


Fig. 13 Time response with and without control (active truss + active cables).

displays a typical frequency-response function (between a force applied in the middle of the truss and an accelerometer located at the top) for various operating conditions: 1) open-loop (no control), 2) active truss, passive cables, 3) active cables, passive truss, and 4) active truss + active cables.

Figure 12d shows that the combined use of an active truss and the active cables eliminates completely the resonant peaks of the frequency response.

Figure 13 shows a typical time response for case (d), with and without control, after an impulsive load. (The control is turned on after 8 s.) The decay rate of the controlled structure is spectacular.

VI. Conclusion

The use of tension cables for active damping of trusses has been investigated. An approximate linear theory for predicting the closed-loop poles has been developed and confirmed by numerical studies and a laboratory experiment. Taking the example of the JPL MPI Testbed, the potential of the proposed methodology to damp large space trusses has been evaluated.

The following remarks are appropriate:

1) The decentralized control approach used here is very simple and can be implemented easily with analog electronics.

2) In theory, the control law has guaranteed stability if one assumes perfect actuator and sensor dynamics. Despite some nonlinearities in the actuator (hysteresis), cable instability has never been observed, but the amount of damping brought to the local cable modes in the absence of sag (0 g) is indeed very small for small amplitudes. As a result, it is clear that, to minimize the detrimental effect of the cable vibration on the structure (mainly due to cable inertia), it is important to use lightweight cables with a pretension such that their first mode is substantially above the frequency range where the structure should be damped. The impact of the residual cables' vibration on microvibrations of the main structure remains to be investigated in more detail. Experiments with a free-floating, laboratory-scale mockup of geometry similar to that of Fig. 7 is underway.

3) The cable network used here for active damping could also be used in a more general HAC-LAC strategy for shape and vibration control of large trusses. This will be the topic of future investigations.

Finally, the technology presented here for large space structures also has promising applications in civil engineering.

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